

Bouncing Palatini cosmologies and their perturbations

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(Dated: April 30, 2010)

Nonsingular cosmologies are investigated in the framework of $f(R)$ gravity within the first order formalism. General conditions for bounces in isotropic and homogeneous cosmology are presented. It is shown that only a quadratic curvature correction is needed to predict a bounce in a flat or to describe cyclic evolution in a curved dust-filled universe. Formalism for perturbations in these models is set up. In the simplest cases, the perturbations diverge at the turnover. Conditions to obtain smooth evolution are derived.

PACS numbers: 04.50.Kd, 98.80.-k, 98.80.Qc.

I. INTRODUCTION

Singularities occurring in General Relativity (GR) may be avoided in more fundamental frameworks where GR predictions are recovered as the low energy limit. Such frameworks could be provided by String or M-theory, or loop quantum gravity. In particular, the evolution of the Big Bang cosmology might be extended to a preceding contracting phase which, due to new physics relevant at high curvature or energy scales, turns into the expanding phase our universe is experiencing now [1, 2], thus avoiding the Big Bang singularity (which in the inflationary picture [3] is manifest rather as geodesic incompleteness [4] than divergence of curvature invariants). These scenarios are called bouncing cosmologies [5].

Bouncing cosmologies can solve the horizon problem, but to replace inflation they should, among other things, also predict a viable, nearly scale invariant spectrum of perturbations. This tricky issue can be circumvented if a curvaton field is responsible for the generation of fluctuations [6]. Otherwise, matching conditions are often required to track the evolution of the perturbations across the bounce [7]. It has been noted that the curvature perturbation may become singular at the bounce, while the gravitational potential, whose growing mode usually persists in the post-bounce era, may have regular behavior [8]. A general solution for the perturbations supports these conclusions [9]. It is also well known that the features of the spectrum can depend sensitively upon the details of the dynamics of the bounce and the physics behind it.

Hence it is useful to consider explicit examples which allow one to scrutinize the possible behaviors of fluctuations at the bounce. However, it is necessary to violate energy conditions (EC). At least the strong EC must be broken to change the sign of the expansion rate, and the null EC cannot be respected if there is no curvature. This rather generically introduces pathologies that, though one may interpret them as only a shortcoming of the effective theory, hinder from reaching definite conclusions

of the evolution of the spectra [10–12]. An example is the perturbation divergence in pre-Big Bang cosmology [13] which can be shown to be an indication of an appearance of a ghost [14]. To avoid EC violating matter fields, one can contemplate modifications of gravity that introduce no new degrees of freedom. This can be achieved with an action involving an infinite series of d'Alembertians acting on the curvature invariants in such a way that the propagator has no poles; these string-inspired nonlocal models¹ have been shown to be ghost-free and asymptotically free at their Newtonian limit [25, 26].

In the present paper we consider the simpler case of second order $f(R)$ gravity, which has similar desirable features. The fourth order metric $f(R)$ models correspond to scalar tensor theories of the form $\mathcal{L}_{4th} = \phi R + V(\phi)$. By erasing the kinetic term implicit in the nonminimal coupling, one obtains the second order theory $\mathcal{L}_{2nd} = \phi R - \frac{3}{4\phi}(\partial\phi)^2 + V(\phi)$. Though this can be problematic in view of the well-posedness of the Cauchy problem [27], these simple models may avoid generic instabilities present in higher order theories [28] and thus provide an effective description of low energy effects of quantum gravity. In particular, loop quantum gravity is expected to modify the cosmological dynamics at high curvatures without introducing new degrees of freedom. To obtain the quadratic density correction appearing in the particular scalar loop quantum cosmology, one may need to consider an infinite number of terms in the potential $V(\phi)$, which may be interpreted to reflect the nonlocal nature of the underlying theory as discussed in Ref.[29]. Note also the recent extension of the framework [30].

Low curvature corrections in these so called Palatini- $f(R)$ theories have also been considered as alternatives to dark energy [31, 32], but though they may produce viable background expansion, they generically fail to pro-

¹ A biscalar-tensor model [15, 16] motivated by the nonlocal cosmology based on inverse d'Alembertian gravity [17, 18] may also accommodate bounces [19], though their viability remains to be shown [20]. Bounces in modified gravity were considered also in e.g. [21–24].

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duce the observed matter power spectrum, at least for pressureless dust cosmology [33–35]. Problems may appear also at microscopic level, as discussions of electron-electron scattering and Hydrogen atoms, seem to imply [36, 37], see also [38, 39]. This may be due to need to reconsider the averaging problem in these models [40], or the coupling of gravity to matter taking torsion and nonmetricity into account [41–43]. One may adopt the approach of considering the formalism as an effective macroscopic description, and then new phenomenology can emerge from the potentially viable high curvature corrections. Studies of spherically symmetric systems show that the classic Solar system tests are passed by these models [44], while the high curvature effects have interesting predictions for white dwarfs and neutron stars [45]. Bounces have been suggested too [46].

We briefly review the first order formalism approach to nonlinear curvature gravity in section II where we also write and solve the cosmological background equations taking into account spatial curvature. We derive the conditions for bounces to occur and confirm them numerically. In section III we present the equations governing the evolution of perturbations in convenient forms, based on derivations in Refs.[47, 48]. Corrections to and generalizations of previous literature are pointed for both the background and the fluctuation equations. In section IV we discuss these results and their implications.

II. BOUNCING BACKGROUNDS IN PALATINI-F(R) GRAVITY

After writing the general equations for the generalized gravity model, we derive the bouncing conditions and analyze them both analytically and numerically.

A. Palatini approach to generalized gravity

Consider gravity theories represented by the action

$$S = \int d^n x \sqrt{-g} \left[\frac{1}{2} f(g^{\mu\nu} \hat{R}_{\mu\nu}) + \mathcal{L}_m(g_{\mu\nu}, \phi, \dots) \right]. \quad (1)$$

Here ϕ, \dots are some matter fields. In the Palatini approach one lets the torsionless connection $\hat{\Gamma}_{\beta\gamma}^\alpha$ vary independently of the metric. The Ricci tensor is constructed solely from this connection,

$$\hat{R}_{\mu\nu} \equiv \hat{\Gamma}_{\mu\nu,\alpha}^\alpha - \hat{\Gamma}_{\mu\alpha,\nu}^\alpha + \hat{\Gamma}_{\alpha\lambda}^\alpha \hat{\Gamma}_{\mu\nu}^\lambda - \hat{\Gamma}_{\mu\lambda}^\alpha \hat{\Gamma}_{\alpha\nu}^\lambda. \quad (2)$$

The field equations which follow from extremization of the action Eq.(1) with respect to metric variations, can be written as

$$F R_\nu^\mu - \frac{1}{2} f \delta_\nu^\mu = T_\nu^\mu, \quad (3)$$

where we have defined $F \equiv \partial f / \partial R$. In GR, $(R - 2\Lambda)/8\pi G$, so $F = 1/8\pi G$. By varying the action with

respect to $\hat{\Gamma}_{\beta\gamma}^\alpha$, obtains

$$\hat{\nabla}_\alpha [\sqrt{-g} g^{\beta\gamma} F] = 0, \quad (4)$$

implying that this connection is compatible with the conformal metric

$$\hat{g}_{\mu\nu} \equiv F^{2/(n-2)} g_{\mu\nu}. \quad (5)$$

This connection governs how the tensor $R_{\mu\nu}$ appearing in the action settles itself, but it turns out that the metric connection determines the geodesics that freely falling particles follow, since the energy momentum

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} \mathcal{L}_m)}{\delta(g^{\mu\nu})}. \quad (6)$$

is conserved according to this connection,

$$\nabla_\mu T_\nu^\mu = 0, \quad (7)$$

whereas in general $\hat{\nabla}_\mu T_\nu^\mu \neq 0$. Therefore we have a metric theory of gravity [49] in the sense of Ref.[50]. The trace of the field equations allows us to solve R as an algebraic function of the matter trace $T \equiv g^{\mu\nu} T_{\mu\nu}$. This central relation reads

$$F R - 2f = T. \quad (8)$$

From now on we set the spacetime dimension to $n = 4$ and use units $8\pi G \equiv c \equiv 1$. Written in the form of GR plus correction terms, the field equations read:

$$G_\nu^\mu(g) = T_\nu^\mu + (1 - F) R_\nu^\mu(g) - \frac{3}{2F} (\nabla^\mu F) (\nabla_\nu F) + \nabla^\mu \nabla_\nu F + \frac{1}{2} \left[(f - R) + \left(1 - \frac{3}{F}\right) \square F + \frac{3}{2F} (\partial F)^2 \right] \delta_\nu^\mu.$$

Since the corrections can be expressed as functions of the matter trace, one can view Eq.(9) as GR with generalised coupling to matter: only the way that "matter tells spacetime how to curve" is modified. So, the whole RHS may be regarded as an effective matter energy-momentum tensor. In vacuum it reduces to a cosmological constant [51]. This is also the case in the presence of conformal matter, i.e. if $T = 0$.

B. Background cosmology

In the spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe with the line element

$$ds^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - Kr^2} + r^2 d\Sigma^2 \right), \quad (9)$$

and a perfect fluid source with a constant equation of state $w = p/\rho$, the Friedmann equation can be written as

$$6F \left(H + \frac{\dot{F}}{2FH} \right)^2 = \rho + 3p + f - 6F \frac{K}{a^2}. \quad (10)$$

In the Appendix A we write the general Friedmann equation when $\dot{w} = 0$ is not assumed. The Hubble parameter can be expressed fully in terms of the curvature scalar, and in our case then rewritten as

$$H^2 = \frac{1}{(1-3w)F} \frac{3(1+w)f - (1+3w)FR - 6F\frac{K}{a^2}}{\left(1 - \frac{3}{2}(1+w)\frac{F'(RF-2f)}{F(RF'-F)}\right)^2}. \quad (11)$$

We have expressed the Hubble rate as a function of R , which we in turn may solve from the trace equation (8). If the scale factor is monotonic, one may find its evolution algebraically once the given matter content as outlined in Ref. [52]. In our case however it is preferable to solve the numerical system by the integrating differential equations, which are shown in the appendix.

C. Bouncing solutions

In general, a necessary condition for a bounce to occur is obtained from (11) and (8) as

$$F = 0, \quad \text{or} \quad (12)$$

$$F(12\frac{K}{a^2} - R) = 3\rho(1+w). \quad (13)$$

Now, by solving the trace equation (8), FR is given by an inverse function of the density, and its form is very dependent of $f(R)$. The quadratic term, which may be considered the leading correction² to GR, can already lead to bouncing cosmology. Let us thus consider the case

$$f(R) = R + \alpha R^2, \quad (14)$$

to study explicitly the background behavior. This model results in a symmetric bounce (the pre-Big Bang is the time reversal of the post-Big Bang evolution). The Friedmann equation is now

$$3H^2 = \frac{a^3 + 2\alpha R_0}{2a^3(a^3 - \alpha R_0)^2} [2a^3 R_0 - 6Ka(a^3 + 2\alpha R_0) + \alpha R_0^2], \quad (15)$$

where $R_0 > 0$ is a constant which is equal to the matter density at $a = 1$. The derivatives of the Hubble rate are written in the appendix as (A3) and (A4). The bounce condition (12,13) now becomes

$$a^3 + 2\alpha R_0 = 0, \quad \text{or} \quad (2a^3 + \alpha R_0)R_0 = 6Ka(a^3 + 2\alpha R_0). \quad (16)$$

In the flat case, the second condition is simply $a^3 = -\alpha R_0/2$, which can be satisfied given a negative $\alpha < 0$.

However, the first condition will be saturated earlier when the scale factor has contracted to $a^3 = -2\alpha R_0$. Then, at the bounce we have $F \rightarrow 0$.

Let us consider whether one may avoid $F = 0$ at the turnover in curved models with $K \neq 0$. Since the normalization of the scale factor is arbitrary, let us assume here that the second bounce condition is fulfilled before the first one at $a = 1$. This implies that

$$2\alpha R_0 > -1, \quad (17)$$

and that

$$R_0 = 6K \frac{1 + 2\alpha R_0}{2 + \alpha R_0}. \quad (18)$$

Solving α from the second constraint and plugging into the first condition gives

$$\frac{4(3K - R_0)}{R_0 - 12K} > -1. \quad (19)$$

We should assume $R_0 > 12K$, since in any realistic universe the curvature is subdominant to the matter density at early times by many orders of magnitude. Then the constraint reduces to $R_0 < 0$, in contradiction to our assumptions. If we however allow a positive curvature to dominate over matter density at the bounce, $K > R_0/12$, we can realize bounces where F stays finite at the turning point. This could have occurred if there was significant amount of inflation after the bounce which diluted away both the curvature and matter density. However, the details of such a case are not of interest to us, as there also the possible signatures from bounce were most probably erased.

As we will see in the following, the $F = 0$ bounce results in divergence of perturbations. In general, assuming negligible curvature but allowing general $f(R)$ and w , the condition for the second type of bounce (13) assumes the very simple form

$$F > 0, \quad f(R) = -\rho(1+3w). \quad (20)$$

In the simplest models considered here in detail, this is a necessary condition for perturbations to stay regular at the bounce. In section IV we briefly discuss possibly viable generalizations of the models.

III. PERTURBATIONS

In the following we first specify our perturbation system, then consider the evolution of the perturbations in pressureless matter as a specific example, and its description in terms of the canonical variable. Cosmological perturbation theory is presented in the reviews [53, 54], and applied to this class of generalized gravity theories in Refs. [47, 48].

² In addition, one notes that this is the only case when the trace equation (8) is linear in the sources. One may then contemplate if more general functions $f(R)$ would be, after averaging, effectively described by the quadratic model.

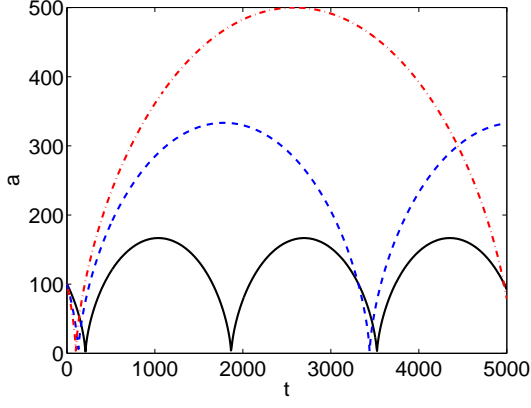


FIG. 1. Solid (black) line: cyclic evolution of the scale factor in the quadratic model with $K > 0$. Dashed (blue) line: the same model with doubly as much matter. Dash-dotted (red) line: the same model with tripled matter density.

A. The perturbation system

The line-element in the perturbed Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime can be written as

$$ds^2 = a^2(t) \left\{ - (1 + 2\phi) dt^2 + b_i dx^i dt + \left[g_{ij}^{(3)} + 2 \left(g_{ij}^{(3)} \psi + h_{ij} \right) \right] dx^i dx^j \right\}. \quad (21)$$

We characterize the scalar perturbations in the longitudinal Newtonian gauge by the variables gravitational potentials ϕ and ψ . Vector perturbations introduce two more degrees of freedom, encoded here into the divergenceless 3-vector field b_i . Gravitational waves are described by the two free components of the symmetric, transverse and traceless 3-tensor h_{ij} . The comoving spatial background metric $g_{ij}^{(3)}$ reduces to δ_{ij} in a flat universe. The vertical bar indicates a covariant derivative based on the Levi-Civita connection of $g_{ij}^{(3)}$. This metric is used to lower and raise spatial indices $i, j, k \dots$ of the perturbation variables. The components of the energy-momentum tensor for a general fluid is imperfect fluid are

$$T_0^0 = -(\bar{\rho} + \delta\rho), \quad (22)$$

$$T_i^0 = -(\bar{\rho} + \bar{p}) \left(v_{,i} + v_i^{(v)} \right), \quad (23)$$

$$T_j^i = (\bar{p} + \delta p) \delta_j^i + \Pi_j^i. \quad (24)$$

Here ρ and p are energy density and pressure, and v , $v^{(v)}$ are the scalar and vector velocity perturbations, respectively. Background quantities are denoted with an overbar, which we will usually omit when unnecessary. The isotropy of the background does not allow anisotropic stress except as a perturbation. This we decompose into the scalar, vector and tensor contributions as

$$\Pi_{ij} \equiv \left(\Pi_{ij}^{(s)} + \frac{1}{3} \Delta \Pi^{(s)} \right) + \Pi_{(ij)}^{(v)} + \Pi_{ij}^{(t)}, \quad (25)$$

where Δ stands for the three-space Laplacian based on the Levi-Civita connection of $g_{ij}^{(3)}$. The vector $\Pi_i^{(v)}$ is divergence-free and the tensor $\Pi_{ij}^{(t)}$ is symmetric, transverse, and traceless. This completes our specification of the perturbation system.

B. Fluid quantities

Next we will discuss the evolution of the system in terms of fluid variables. First we consider the density perturbation in the comoving gauge (i.e. CDM rest frame in the present case) and then the velocity perturbation in the uniform-density gauge (i.e. the frame where CDM is smoothly distributed). The former quantity becomes ill-defined at the bounce, the latter behaves somewhat better.

It is convenient to introduce the comoving density perturbation Δ which is given by longitudinal gauge quantities as follows:

$$\Delta = \delta + 3H(1+w) \frac{av}{k}, \quad (26)$$

The evolution equation for the perturbations has been derived in the general case and is of the form

$$\ddot{\Delta} = D_1 H \dot{\Delta} + \left(D_2 H^2 + D_k \frac{k^2}{a^2} \right) \Delta + P_1 H \dot{\Pi} + P_2 H^2 \Pi, \quad (27)$$

where the dimensionless coefficients are given in the appendix of Ref. [48]. In the case of pressureless dust the evolution equation simplifies to

$$\ddot{\Delta} + (2H + \mathcal{F}) \dot{\Delta} = \left(\frac{\ddot{F}}{H} + 2\dot{H} + \frac{\dot{H}}{H} \mathcal{F} - \frac{k^2}{a^2} c_{eff}^2 \right) \Delta. \quad (28)$$

where we have defined the auxiliary quantity

$$\mathcal{F} \equiv \frac{2}{\dot{F} + 2FH} \left(\ddot{F} - \frac{\dot{F}^2}{F} - \frac{\dot{F}\dot{H}}{H} \right). \quad (29)$$

and the effective sound speed squared

$$c_{eff}^2 \equiv \frac{\dot{F}}{3(\dot{F} + 2FH)}. \quad (30)$$

If both \mathcal{F} and c_{eff}^2 vanish GR evolution is recovered, so these variables represent the modified gravity effects. In the specific example model discussed in section II C, both of these terms apparently diverge at the bounce. In particular, we have divisions by H , $\dot{F} + 2FH$ and F , where the first term vanishes always, the second term vanishes at least for dust, and the last term vanishes at least for the flat dust bounce model. In particular, for the quadratic model

$$c_S^2 = -\frac{\alpha R_0}{a^3 - \alpha R_0}, \quad (31)$$

$$\mathcal{F} = \frac{18H\alpha R_0 a^3}{(a^3 - \alpha R_0)(a^3 + 2\alpha R_0)}, \quad (32)$$

and we see that when $a^3 = -\alpha R_0$, the sound speed in fact is regular but the factor \mathcal{F} is not. The \dot{H} and \ddot{H}/H are given in the appendix as (A3) and (A4). Though the comoving gauge is where we want our observable density in, this coordinate system can become ill-defined at the bounce. However, a possibility remains that this is not a physical problem.

The perturbations can be carried across the bounce in another variable than the comoving gauge density perturbation. Another convenient quantity to consider is v_δ , the velocity perturbation of matter evaluated in the uniform-density gauge. From Eq.(26), we have

$$v_\delta = \frac{k}{3aH}\Delta, \quad (33)$$

and readily obtain from (27) the evolution equation which coincides with the Eq.(46) in Ref.[47],

$$\ddot{v}_\delta + \left(4H + 2\frac{\dot{H}}{H} + \mathcal{F}\right)\dot{v}_\delta + \left(3(\dot{H} + H^2) + H\mathcal{F} + \frac{k^2}{a^2}c_{eff}^2\right)v_\delta = 0. \quad (34)$$

The divergent $1/H$ terms in the second expression cancel,

$$4H + 2\frac{\dot{H}}{H} + \mathcal{F} = \frac{2}{\dot{F} + 2FH} \left[\ddot{F} - \frac{\dot{F}^2}{F} + 2(FH)^\bullet + 4FH^2 \right]. \quad (35)$$

However, the \dot{F}/F can divergent, when $F \rightarrow 0$ in the flat bounce model. This can be avoided with curvature $K > \rho/12$, or in the bounce of the type (20) but even then typically $\dot{F} \rightarrow 0$ at the turnover. Consequently, the factor \mathcal{F} remains apparently divergent due to $\mathcal{F} \sim \frac{1}{\dot{F} + 2FH}$. Thus, by considering (33) instead of (26) the apparent divergencies of the coefficients in the evolution equations can be made less severe (from $\sim 1/H^2$ to $\sim H$) but they seem to persist.

C. Canonical variable

The canonical variable ν obeys the equation of motion

$$\ddot{\nu} + H\dot{\nu} + \left(-\frac{3}{4}H^2 + \frac{1}{2}\dot{H} - \frac{K}{a^2} + C_\nu + \frac{k^2 - 3K}{a^2}c_{eff}^2\right)\nu = 0, \quad (36)$$

where

$$C_\nu = -\frac{\dot{F}}{2F} \left(3H + \frac{\dot{F}}{F}\right). \quad (37)$$

The coefficients remain apparently divergent for the flat dust bounce model of section II C, because the potential

becomes infinite due to the $\sim \dot{F}^2/F^2$ term. However, for the curvature-dominated model or bounce of the type (20) both C_ν and the sound speed remain regular at the bounce, since $H, \dot{F} \rightarrow 0$. Thus we have found the explicit conditions for the perturbations to be carried smoothly across the bounce. The relation between the Mukhanov-Sasaki variable ν and the comoving density perturbation is given by

$$\nu(t) = \exp\left\{\frac{1}{2}\int^t [H(t') + \mathcal{F}(t')] dt'\right\}\delta(t). \quad (38)$$

Hence, for a model with F not dipping to zero, one may use the regular equation (36) to solve the evolution, and the transformation (38) to obtain the results in terms of the observables.

IV. CONCLUSIONS

Within the Palatini framework one may consider extensions of GR without introducing new degrees of freedom. Hence they may serve as useful toy models describing more completely the cosmological evolution, with motivations from e.g. loop quantum cosmology. We showed that there are nonsingular bouncing backgrounds in simple examples of such quantum corrected gravity models, and set up the formalism for the perturbations in these models in order to monitor the evolution of their spectra across a bounce.

The models of the type (12), characterized by $F \rightarrow 0$ at the bounce, were found to feature singular behavior of perturbations in a flat, dust-filled universe. This may be cured in the curvature-dominated case, reflecting the fact that in the $K = 0$ case the effective matter sources necessarily violate the null EC, whereas this is not the case if curvature is present. Since F gives the sign of the graviton action, pathologies were to be expected at $F \rightarrow 0$. At this point the conformal relation between Einstein and Jordan frames (5) becomes ill-defined. Let us note though that as the perturbations explode, their backreaction will render the perturbative system invalid, and as these nonlinear effects are very difficult to tackle in practice, we cannot say if there is a true singularity or not and whether the bounce occurs or not. In this light the problem is rather unpredictivity.

Obviously, it would be interesting to explore possible ways to obtain bounces of the type (20). Such might be constructed by considering just more general functions $f(R)$ than the one involving solely a monomial correction.

Another way to obtain smooth evolution could be to include more general sources than completely pressureless fluids. Apart from allowing $w \neq 0$, one may also consider stabilizing the system with entropic or anisotropic stresses. Indeed, this has previously proved successful in eliminating instabilities in matter perturbations in some dark energy models [48, 55] (however, those models based on infrared gravity modifications may be otherwise prob-

lematical as discussed in the introduction). More realistically, also radiation would be included as a source and this changes the dynamics and possibly the conclusions. Our results can be directly applied to such more general models with possibly regular evolution. This is left for future studies.

As a concluding remark we note it is possible that a complete evolution of the background and structures of the universe is not amenable to classical description by second order differential equations, and one may have to take into account in a more nontrivial way the presently unknown physics at the very high curvature scales in order to provide a fully consistent coarse-grained picture of the cosmology that emerges. Meanwhile, the quest for the effective field equations for gravitational interactions, at both high and low curvature regimes, is ongoing.

Appendix A: Friedmann equations

1. General case

Without assuming a constant equation of state or vanishing curvature, the Hubble parameter may be written as

$$H = \quad (A1)$$

$$\dot{H} = - \frac{a^7 R_0 (a^2 - 24\alpha K) - 2a^{10} K + 6a^4 \alpha R_0^2 (a^2 - 6\alpha K) + a\alpha^2 R_0^3 (3a^2 + 8\alpha K) - \alpha^3 R_0^4}{2(a^4 - a\alpha R_0)^3}. \quad (A3)$$

$$\begin{aligned} \frac{\ddot{H}}{H} = & \frac{1}{2(a^4 - a\alpha R_0)^4} \left[a^{10} R_0 (3a^2 - 134\alpha K) - 4a^{13} K \right. \\ & \left. + 6a^7 \alpha R_0^2 (7a^2 - 64\alpha K) + a^4 \alpha^2 R_0^3 (45a^2 + 52\alpha K) - 4a\alpha^3 R_0^4 (3a^2 + 4\alpha K) + 3\alpha^4 R_0^5 \right]. \end{aligned} \quad (A4)$$

Eq.(A3) is a suitable form for numerical integration. We checked that the solution gives Eq.(15) and that its numerical derivative reproduces Eq.(A4). The latter algebraic expression is needed specifically at the bounce where $\ddot{H}, H \rightarrow 0$.

$$\frac{9\dot{w}F' + (F - F'R)\sqrt{3F(F(R - 12\frac{K}{a^2}) + 3\rho(1+w))}}{3(2F^2 - 2FF'R - 3F'\rho\alpha_w)}.$$

where $\alpha_w = (1+w)(1-3w)$ and $F' = dF/dR$. For a constant equation of state $\dot{w} = 0$, this reduces to

$$3H^2 = \frac{F[3\rho(1+w) - F(12\frac{K}{a^2} - R)]}{4\left[F + \frac{3}{2}\frac{F'\rho(1+w)(1-3w)}{RF' - F}\right]^2}. \quad (A2)$$

In the limit $K = 0$, this agrees with the formulas in the references [32, 47, 52] (and in several later references). However, this does not reduce to the Hubble law considered in Ref.[46], and consequently our solutions and results in subsection II C are somewhat different from theirs. For example, their Eq.(20) for the specific case of the quadratic model is quite different from our Eq.(15) also when $K = 0$. Note that in Ref.[56] the authors have corrected a mistake regarding nonzero spatial curvature in Ref.[46], and there the considerations are in accordance with ours here.

2. Quadratic model

The first two derivatives of the Hubble rate in the quadratic model (14) are

ACKNOWLEDGMENTS

The author is grateful to T. Biswas and A. Mazumdar for enlightening discussions and to G. Olmo, P. Peter and T. Sotiriou for useful comments on the manuscript. This work was supported by the FOM and the Finnish Academy.

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